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Second-Order Parabolic Partial Differential Equations > Nonlinear Heat Equation of General Form

$$9. \quad \frac{\partial w}{\partial t} = \frac{\partial}{\partial x} \left[f(w) \frac{\partial w}{\partial x} \right].$$

Nonlinear heat equation of general form. This equation occurs in nonlinear problems of heat and mass transfer and flows in porous media.

1°. Traveling-wave solution in implicit form:

$$k^2 \int \frac{f(w) dw}{\lambda w + C_1} = kx + \lambda t + C_2,$$

where C_1 , C_2 , k , and λ are arbitrary constants. To $\lambda = 0$ there corresponds a stationary solution.

2°. Self-similar solution:

$$w = w(z), \quad z = xt^{-1/2},$$

where the function $w(z)$ is determined by the ordinary differential equation $[f(w)w'_z]'_z + \frac{1}{2}zw'_z = 0$.

See also special cases of the nonlinear heat equation:

- [heat equation with a power-law nonlinearity](#) ,
- [heat equation with an exponential nonlinearity](#) .

References

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