1. \( \frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial x^2} + w \frac{\partial w}{\partial x}. \)

**Burgers equation.** It is used for describing wave processes in acoustics and hydrodynamics.

1°. Solutions:

\[
\begin{align*}
  w(x, t) &= \lambda + \frac{2}{x + \lambda t + A}, \\
  w(x, t) &= \frac{4x + 2A}{x^2 + Ax + 2t + B}, \\
  w(x, t) &= \frac{6(x^2 + 2t + A)}{x^3 + 6xt + 3Ax + B}, \\
  w(x, t) &= \frac{2\lambda}{1 + A \exp(-\lambda^2 t - \lambda x)}, \\
  w(x, t) &= -\lambda + \frac{A \exp[A(x - \lambda t)] - B}{\exp[A(x - \lambda t)] + B},
\end{align*}
\]

where \( A, B, \) and \( \lambda \) are arbitrary constants.

2°. Other solutions can be obtained using the following formula (Hopf–Cole transformation):

\[ w(x, t) = \frac{2}{u} \frac{\partial u}{\partial x}, \]

where \( u = u(x, t) \) is a solution of the linear heat equation, \( u_t = u_{xx}. \)

3°. The Cauchy problem with the initial condition:

\[ w = f(x) \quad \text{at} \quad t = 0, \quad -\infty < x < \infty. \]

Solution:

\[
\begin{align*}
  w(x, t) &= 2 \frac{\partial}{\partial x} \ln F(x, t), \\
  F(x, t) &= \frac{1}{\sqrt{4\pi t}} \int_{-\infty}^{\infty} \exp \left[ -\frac{(x - \xi)^2}{4t} - \frac{1}{2} \int_{0}^{\xi} f(\xi') \, d\xi' \right] d\xi.
\end{align*}
\]

**References**


Burgers Equation

Copyright © 2004 Andrei D. Polyanin