3. $\dfrac{\partial w}{\partial t} = \dfrac{1}{x^n}\dfrac{\partial}{\partial x}\left[x^n f(w)\dfrac{\partial w}{\partial x}\right] + g(w).$

This is a nonlinear equation of heat and mass transfer in the radial symmetric case ($n = 1$ corresponds to a plane problem and $n = 2$ to a spatial one).

1°. Let $f(w)$ and $g(w)$ be defined by

$$f(w) = w\varphi'_w(w), \qquad g(w) = a(n+1)w + 2a\frac{\varphi(w)}{\varphi'_w(w)},$$

where $\varphi(w)$ is an arbitrary function. In this case, there is a functional separable solution defined implicitly by

$$\varphi(w) = Ce^{2at} - \tfrac{1}{2}ax^2,$$

where C is an arbitrary constant.

2°. Let $f(w)$ and $g(w)$ be defined as follows:

$$f(w) = a\varphi^{-\frac{n+1}{2}}\varphi' \int \varphi^{\frac{n+1}{2}}\,dw, \qquad g(w) = b\frac{\varphi}{\varphi'},$$

where $\varphi = \varphi(w)$ is an arbitrary function. In this case, there is a functional separable solution defined implicitly by

$$\varphi(w) = \frac{bx^2}{Ce^{-bt} - 4a}.$$

3°. Let $f(w)$ and $g(w)$ be defined in the formulas

$$f(w) = A\frac{V(z)}{V'_z(z)}, \qquad g(w) = B\left[2z^{-\frac{n+1}{2}}V'_z(z) + (n+1)z^{-\frac{n+3}{2}}V(z)\right], \qquad (1)$$

where $V(z)$ is an arbitrary function of z, A and B are arbitrary constants ($AB \neq 0$), and the function $z = z(w)$ is determined implicitly by

$$w = \int z^{-\frac{n+1}{2}}V'_z(z)\,dz + C_1; \qquad (2)$$

C_1 is arbitrary constant. Then, there is a functional separable solution of the form (2) where

$$z = -\frac{x^2}{4At + C_2} + 2Bt + \frac{BC_2}{2A},$$

and C_2 is an arbitrary constant.

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