3. \[ \frac{\partial w}{\partial t} = \frac{1}{x^n} \frac{\partial}{\partial x} \left[ x^n f(w) \frac{\partial w}{\partial x} \right] + g(w). \]

This is a nonlinear equation of heat and mass transfer in the radial symmetric case \((n = 1)\) corresponds to a plane problem and \(n = 2\) to a spatial one).

1°. Let \(f(w)\) and \(g(w)\) be defined by

\[
 f(w) = w \varphi'_w(w), \quad g(w) = a(n + 1)w + 2a \frac{\varphi(w)}{\varphi'_w(w)},
\]

where \(\varphi(w)\) is an arbitrary function. In this case, there is a functional separable solution defined implicitly by

\[
 \varphi(w) = C e^{2at} - \frac{1}{a} x^2,
\]

where \(C\) is an arbitrary constant.

2°. Let \(f(w)\) and \(g(w)\) be defined as follows:

\[
 f(w) = a \varphi^{\frac{n+1}{2}} \varphi' \int \varphi^{\frac{n+1}{2}} dw, \quad g(w) = b \frac{\varphi}{\varphi'},
\]

where \(\varphi = \varphi(w)\) is an arbitrary function. In this case, there is a functional separable solution defined implicitly by

\[
 \varphi(w) = \frac{bx^2}{Ce^{4at} - 4a}.
\]

3°. Let \(f(w)\) and \(g(w)\) be defined in the formulas

\[
 f(w) = A \frac{V(z)}{V'_z(z)}, \quad g(w) = B \left[ 2z^{-\frac{n+1}{2}} V'_z(z) + (n + 1)z^{-\frac{n+3}{2}} V(z) \right],
\]

where \(V(z)\) is an arbitrary function of \(z\), \(A\) and \(B\) are arbitrary constants \((AB \neq 0)\), and the function \(z = z(w)\) is determined implicitly by

\[
 w = \int z^{-\frac{n+1}{2}} V'_z(z) dz + C_1; \quad (2)
\]

\(C_1\) is arbitrary constant. Then, there is a functional separable solution of the form \((2)\) where

\[
 z = -\frac{x^2}{4At + C_2} + 2Bt + \frac{BC_2}{2A},
\]

and \(C_2\) is an arbitrary constant.

References

