1. $\dfrac{\partial^2 w}{\partial t^2} = \dfrac{\partial^2 w}{\partial x^2} + aw + bw^n.$

Klein–Gordon equation with a power-law nonlinearity - 1.

1°. Traveling-wave solutions for $a > 0$:

$$w(x,t) = \left[\frac{2b\sinh^2 z}{a(n+1)}\right]^{\frac{1}{1-n}}, \quad z = \tfrac{1}{2}\sqrt{a}\,(1-n)(x\sinh C_1 \pm t\cosh C_1) + C_2 \quad \text{if } b(n+1) > 0,$$

$$w(x,t) = \left[-\frac{2b\cosh^2 z}{a(n+1)}\right]^{\frac{1}{1-n}}, \quad z = \tfrac{1}{2}\sqrt{a}\,(1-n)(x\sinh C_1 \pm t\cosh C_1) + C_2 \quad \text{if } b(n+1) < 0,$$

where C_1 and C_2 are arbitrary constants.

2°. Traveling-wave solutions for $a < 0$ and $b(n+1) > 0$:

$$w(x,t) = \left[-\frac{2b\cos^2 z}{a(n+1)}\right]^{\frac{1}{1-n}}, \quad z = \tfrac{1}{2}\sqrt{|a|}\,(1-n)(x\sinh C_1 \pm t\cosh C_1) + C_2.$$

3°. For $a = 0$, there is a self-similar solution of the form $w = t^{\frac{2}{1-n}} F(z)$, where $z = x/t$.

4°. For other exact solutions of this equation, see the nonlinear Klein–Gordon equation with $f(w) = aw + bw^n$.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Nonlinear Partial Differential Equations* , Chapman & Hall/CRC, Boca Raton, 2004.

