3. $\dfrac{\partial^2 w}{\partial t^2} = a^2 \dfrac{\partial^2 w}{\partial x^2} + b e^{\beta w}$.

Modified Liouville equation.

1°. Traveling-wave solutions:

$$w(x,t) = \frac{1}{\beta} \ln\left[\frac{2(B^2 - a^2 A^2)}{b\beta (Ax + Bt + C)^2}\right],$$
$$w(x,t) = \frac{1}{\beta} \ln\left[\frac{2(a^2 A^2 - B^2)}{b\beta \cosh^2(Ax + Bt + C)}\right],$$
$$w(x,t) = \frac{1}{\beta} \ln\left[\frac{2(B^2 - a^2 A^2)}{b\beta \sinh^2(Ax + Bt + C)}\right],$$
$$w(x,t) = \frac{1}{\beta} \ln\left[\frac{2(B^2 - a^2 A^2)}{b\beta \cos^2(Ax + Bt + C)}\right],$$

where A, B, and C are arbitrary constants.

2°. Functional separable solutions:

$$w(x,t) = \frac{1}{\beta} \ln\left(\frac{8a^2 C}{b\beta}\right) - \frac{2}{\beta} \ln\left|(x+A)^2 - a^2(t+B)^2 + C\right|,$$
$$w(x,t) = -\frac{2}{\beta} \ln\left[C_1 e^{\lambda x} \pm \frac{\sqrt{2b\beta}}{2a\lambda} \sinh(a\lambda t + C_2)\right],$$
$$w(x,t) = -\frac{2}{\beta} \ln\left[C_1 e^{\lambda x} \pm \frac{\sqrt{-2b\beta}}{2a\lambda} \cosh(a\lambda t + C_2)\right],$$
$$w(x,t) = -\frac{2}{\beta} \ln\left[C_1 e^{a\lambda t} \pm \frac{\sqrt{-2b\beta}}{2a\lambda} \sinh(\lambda x + C_2)\right],$$
$$w(x,t) = -\frac{2}{\beta} \ln\left[C_1 e^{a\lambda t} \pm \frac{\sqrt{2b\beta}}{2a\lambda} \cosh(\lambda x + C_2)\right],$$

where A, B, C, C_1, C_2, and λ are arbitrary constants.

3°. General solution:

$$w(x,t) = \frac{1}{\beta}\left[f(z) + g(y)\right] - \frac{2}{\beta} \ln\left|k \int \exp\left[f(z)\right] dz - \frac{b\beta}{8a^2 k} \int \exp\left[g(y)\right] dy\right|,$$
$$z = x - at, \qquad y = x + at,$$

where $f = f(z)$ and $g = g(y)$ are arbitrary functions and k is an arbitrary constant.

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