4. $\dfrac{\partial^2 w}{\partial t^2} = \dfrac{\partial^2 w}{\partial x^2} + ae^{\beta w} + be^{2\beta w}.$

Klein–Gordon equation with a exponential nonlinearity.

1°. Traveling-wave solutions:

$$w(x,t) = -\frac{1}{\beta}\ln\left[\frac{a\beta}{C_1^2 - C_2^2} + C_3\exp(C_1 x + C_2 t) + \frac{a^2\beta^2 + b\beta(C_1^2 - C_2^2)}{4C_3(C_1^2 - C_2^2)^2}\exp(-C_1 x - C_2 t)\right],$$

$$w(x,t) = -\frac{1}{\beta}\ln\left[\frac{a\beta}{C_2^2 - C_1^2} + \frac{\sqrt{a^2\beta^2 + b\beta(C_2^2 - C_1^2)}}{C_2^2 - C_1^2}\sin(C_1 x + C_2 t + C_3)\right],$$

where C_1, C_2, and C_3 are arbitrary constants.

2°. For other exact solutions of this equation, see the nonlinear Klein–Gordon equation with $f(w) = ae^{\beta w} + be^{2\beta w}$.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Nonlinear Partial Differential Equations* , Chapman & Hall/CRC, Boca Raton, 2004.

