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$$4. \quad \frac{\partial^2 w}{\partial t^2} = \frac{\partial^2 w}{\partial x^2} + a e^{\beta w} + b e^{2\beta w}.$$

Klein–Gordon equation with a exponential nonlinearity.

1°. Traveling-wave solutions:

$$w(x, t) = -\frac{1}{\beta} \ln \left[\frac{a\beta}{C_1^2 - C_2^2} + C_3 \exp(C_1 x + C_2 t) + \frac{a^2 \beta^2 + b\beta(C_1^2 - C_2^2)}{4C_3(C_1^2 - C_2^2)^2} \exp(-C_1 x - C_2 t) \right],$$

$$w(x, t) = -\frac{1}{\beta} \ln \left[\frac{a\beta}{C_2^2 - C_1^2} + \frac{\sqrt{a^2 \beta^2 + b\beta(C_2^2 - C_1^2)}}{C_2^2 - C_1^2} \sin(C_1 x + C_2 t + C_3) \right],$$

where C_1 , C_2 , and C_3 are arbitrary constants.

2°. For other exact solutions of this equation, see the [nonlinear Klein–Gordon equation](#) with $f(w) = a e^{\beta w} + b e^{2\beta w}$.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Nonlinear Partial Differential Equations*, Chapman & Hall/CRC, Boca Raton, 2004.

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<http://eqworld.ipmnet.ru/en/solutions/npde/npde2104.pdf>