



$$6. \quad \frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial x} \left(a e^{\lambda x} \frac{\partial w}{\partial x} \right) + f(w), \quad a > 0.$$

This equation describes the propagation of nonlinear waves in an inhomogeneous medium.

Functional separable solution:

$$w = w(z), \quad z = [4k e^{-\lambda x} - ak\lambda^2(t + C)^2]^{1/2}, \quad k = \pm 1,$$

where C is an arbitrary constant and the function $w = w(z)$ is determined by the autonomous ordinary differential equation $ak\lambda^2 w''_{zz} + f(w) = 0$.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Nonlinear Partial Differential Equations*, Chapman & Hall/CRC, Boca Raton, 2004.