7. $\dfrac{\partial^2 w}{\partial t^2} = \dfrac{\partial}{\partial x}\left[f(w)\dfrac{\partial w}{\partial x}\right].$

This equation is encountered in wave and gas dynamics.

1°. Traveling-wave solution in implicit form:

$$\lambda^2 w - \int f(w)\,dw = A(x+\lambda t) + B,$$

where A, B, and λ are arbitrary constants.

2°. Self-similar solution:

$$w = w(\xi), \quad \xi = \frac{x+A}{t+B},$$

where the function $w(\xi)$ is determined by the ordinary differential equation $(\xi^2 w'_\xi)'_\xi = \left[f(w)w'_\xi\right]'_\xi$, which admits the first integral

$$\left[\xi^2 - f(w)\right] w'_\xi = C.$$

To the special case $C = 0$ there corresponds the solution in implicit form: $\xi^2 = f(w)$.

3°. This equation can be reduced to a linear one; see Item 3° in the stationary anisotropic heat equation, where one should set $g(w) = -1$ and $y = t$.

References

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