3. $\dfrac{\partial^2 w}{\partial x^2} + \dfrac{\partial^2 w}{\partial y^2} = ae^{\beta w}.$

This equation occurs in combustion theory and is a special case of equation 3.1.7 with $f(w) = ae^{\beta w}$.

1°. Solutions:

$$w(x,y) = \frac{1}{\beta} \ln\left[\frac{2(A^2+B^2)}{a\beta(Ax+By+C)^2}\right] \qquad \text{if } a\beta > 0,$$

$$w(x,y) = \frac{1}{\beta} \ln\left[\frac{2(A^2+B^2)}{a\beta \sinh^2(Ax+By+C)}\right] \qquad \text{if } a\beta > 0,$$

$$w(x,y) = \frac{1}{\beta} \ln\left[\frac{-2(A^2+B^2)}{a\beta \cosh^2(Ax+By+C)}\right] \qquad \text{if } a\beta < 0,$$

$$w(x,y) = \frac{1}{\beta} \ln\left[\frac{2(A^2+B^2)}{a\beta \cos^2(Ax+By+C)}\right] \qquad \text{if } a\beta > 0,$$

$$w(x,y) = \frac{1}{\beta} \ln\left(\frac{8C}{a\beta}\right) - \frac{2}{\beta} \ln\left|(x+A)^2 + (y+B)^2 - C\right|,$$

where A, B, and C are arbitrary constants. The first four solutions are of traveling-wave type and the last one is a radial symmetric solution with center at the point $(-A, -B)$.

2°. Functional separable solutions:

$$w(x,y) = -\frac{2}{\beta} \ln\left[C_1 e^{ky} \pm \frac{\sqrt{2a\beta}}{2k} \cos(kx + C_2)\right],$$

$$w(x,y) = \frac{1}{\beta} \ln \frac{2k^2(B^2 - A^2)}{a\beta[A\cosh(kx+C_1) + B\sin(ky+C_2)]^2},$$

$$w(x,y) = \frac{1}{\beta} \ln \frac{2k^2(A^2 + B^2)}{a\beta[A\sinh(kx+C_1) + B\cos(ky+C_2)]^2},$$

where A, B, C_1, C_2, and k are arbitrary constants (x and y can be swapped to give another three solutions).

3°. General solution:

$$w(x,y) = -\frac{2}{\beta} \ln \frac{\sqrt{|a|\beta^2}\,\left[1 + \operatorname{sign}(a\beta)\Phi(z)\overline{\Phi(z)}\,\right]}{4|\Phi'_z(z)|},$$

where $\Phi = \Phi(z)$ is an arbitrary analytic (holomorphic) function of the complex variable $z = x + iy$ with nonzero derivative, and the bar over a symbol denotes the complex conjugate.

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