3. $\dfrac{\partial^2 w}{\partial x^2} + \dfrac{\partial^2 w}{\partial y^2} = a e^{\beta w}.$

This equation occurs in combustion theory and is a special case of equation 3.1.7 with $f(w) = ae^{\beta w}$.

1°. Solutions:

$$w(x,y) = \frac{1}{\beta}\ln\left[\frac{2(A^2+B^2)}{a\beta(Ax+By+C)^2}\right] \qquad \text{if } a\beta > 0,$$

$$w(x,y) = \frac{1}{\beta}\ln\left[\frac{2(A^2+B^2)}{a\beta\sinh^2(Ax+By+C)}\right] \qquad \text{if } a\beta > 0,$$

$$w(x,y) = \frac{1}{\beta}\ln\left[\frac{-2(A^2+B^2)}{a\beta\cosh^2(Ax+By+C)}\right] \qquad \text{if } a\beta < 0,$$

$$w(x,y) = \frac{1}{\beta}\ln\left[\frac{2(A^2+B^2)}{a\beta\cos^2(Ax+By+C)}\right] \qquad \text{if } a\beta > 0,$$

$$w(x,y) = \frac{1}{\beta}\ln\left(\frac{8C}{a\beta}\right) - \frac{2}{\beta}\ln\left|(x+A)^2+(y+B)^2-C\right|,$$

where A, B, and C are arbitrary constants. The first four solutions are of traveling-wave type and the last one is a radial symmetric solution with center at the point $(-A,-B)$.

2°. Functional separable solutions:

$$w(x,y) = -\frac{2}{\beta}\ln\left[C_1e^{ky} \pm \frac{\sqrt{2a\beta}}{2k}\cos(kx+C_2)\right],$$

$$w(x,y) = \frac{1}{\beta}\ln\frac{2k^2(B^2-A^2)}{a\beta[A\cosh(kx+C_1)+B\sin(ky+C_2)]^2},$$

$$w(x,y) = \frac{1}{\beta}\ln\frac{2k^2(A^2+B^2)}{a\beta[A\sinh(kx+C_1)+B\cos(ky+C_2)]^2},$$

where A, B, C_1, C_2, and k are arbitrary constants (x and y can be swapped to give another three solutions).

3°. General solution:

$$w(x,y) = -\frac{2}{\beta}\ln\frac{\sqrt{|a|\beta^2}\,\left[1+\operatorname{sign}(a\beta)\Phi(z)\overline{\Phi(z)}\,\right]}{4|\Phi'_z(z)|},$$

where $\Phi = \Phi(z)$ is an arbitrary analytic (holomorphic) function of the complex variable $z = x + iy$ with nonzero derivative, and the bar over a symbol denotes the complex conjugate.

References

Vekua, I. N., Remarks on the properties of solutions to equation $\Delta u = -Ke^{2u}$ [in Russian], *Sib. Matem. Zhurn.*, Vol. 1, No. 3, pp. 331–342, 1960.

Frank-Kamenetskii, D. A., *Diffusion and Heat Transfer in Chemical Kinetics* [in Russian], Nauka, Moscow, 1987.

Zaitsev, V. F. and Polyanin, A. D., *Handbook of Partial Differential Equations: Exact Solutions* [in Russian], Mezhdunarodnaya Programma Obrazovaniya, Moscow, 1996.

Aristov, S. N., Exact periodic and localized solutions of the equation $h_t = \Delta \ln h$, *J. Appl. Mech. & Tech. Phys.*, Vol. 40, No. 1, pp. 16–19, 1999.

Sabitov, I. Kh., On solutions of the equation $\Delta u = f(x,y)e^{cu}$ in some special cases [in Russian], *Mat. Sbornik*, Vol. 192, No. 6, pp. 89–104, 2001.

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Nonlinear Partial Differential Equations* , Chapman & Hall/CRC, Boca Raton, 2004.

