4. $\dfrac{\partial^2 w}{\partial x^2} + \dfrac{\partial^2 w}{\partial y^2} = ae^{\beta w} + be^{2\beta w}.$

1°. Traveling-wave solutions:

$$w(x,y) = -\frac{1}{\beta}\ln\left[-\frac{a\beta}{C_1^2+C_2^2} + C_3\exp(C_1x+C_2y) + \frac{a^2\beta^2 - b\beta(C_1^2+C_2^2)}{4C_3(C_1^2+C_2^2)^2}\exp(-C_1x-C_2y)\right],$$

$$w(x,y) = -\frac{1}{\beta}\ln\left[\frac{a\beta}{C_1^2+C_2^2} + \frac{\sqrt{a^2\beta^2 + b\beta(C_1^2+C_2^2)}}{C_1^2+C_2^2}\sin(C_1x+C_2y+C_3)\right],$$

where C_1, C_2, and C_3 are arbitrary constants.

2°. For other exact solutions of this equation, see equation 3.1.7 with $f(w) = ae^{\beta w} + be^{2\beta w}$.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Nonlinear Partial Differential Equations* , Chapman & Hall/CRC, Boca Raton, 2004.

