5. $\dfrac{\partial^2 w}{\partial x^2}+\dfrac{\partial^2 w}{\partial y^2}=\alpha w\ln(\beta w).$

1°. Solutions:

$$w(x,y)=\frac{1}{\beta}\exp\Big[\tfrac{1}{4}\alpha(x+A)^2+\tfrac{1}{4}\alpha(y+B)^2+1\Big],$$

$$w(x,y)=\frac{1}{\beta}\exp\Big[A(x+B)^2\pm\sqrt{A\alpha-4A^2}\,(x+B)(y+C)+(\tfrac{1}{4}\alpha-A)(y+C)^2+\tfrac{1}{2}\Big],$$

where A, B, and C are arbitrary constants.

2°. There are exact solutions of the following forms:

$$w(x,y)=F(z),\quad z=Ax+By,$$
$$w(x,y)=G(r),\quad r=\sqrt{(x+C_1)^2+(y+C_2)^2},$$
$$w(x,y)=f(x)g(y).$$

References

Shercliff, J. A., Simple rotational flows, *J. Fluid Mech.*, Vol. 82, No. 4, pp. 687–703, 1977.

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Nonlinear Partial Differential Equations* , Chapman & Hall/CRC, Boca Raton, 2004.

