6. $\dfrac{\partial^2 w}{\partial x^2} + \dfrac{\partial^2 w}{\partial y^2} = \alpha \sin(\beta w)$**.**

1°. Functional separable solution for $\alpha = \beta = 1$:

$$w(x,y) = 4\arctan\left(\cot A \frac{\cosh F}{\cosh G}\right), \quad F = \frac{\cos A}{\sqrt{1+B^2}}(x - By), \quad G = \frac{\sin A}{\sqrt{1+B^2}}(y + Bx),$$

where A and B are arbitrary constants.

2°. Functional separable solution (generalizes the solution of Item 1°):

$$w(x,y) = \frac{4}{\beta}\arctan\bigl[f(x)g(y)\bigr],$$

where the functions $f = f(x)$ and $g = g(y)$ are determined by the first-order autonomous ordinary differential equations

$$(f'_x)^2 = Af^4 + Bf^2 + C, \qquad (g'_y)^2 = Cg^4 + (\alpha\beta - B)g^2 + A,$$

and A, B, and C are arbitrary constants.

3°. For other exact solutions of this equation, see equation 3.1.7 with $f(w) = \alpha \sin(\beta w)$.

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