7. $\dfrac{\partial^2 w}{\partial x^2} + \dfrac{\partial^2 w}{\partial y^2} = f(w).$

This is a *stationary heat equation with a nonlinear source*.

1°. Suppose $w = w(x, y)$ is a solution of the equation in question. Then the functions

$$w_1 = w(\pm x + C_1, \pm y + C_2),$$
$$w_2 = w(x \cos\beta - y \sin\beta,\ x \sin\beta + y \cos\beta),$$

where C_1, C_2, and β are arbitrary constants, are also solutions of the equation (the plus or minus signs in w_1 are chosen arbitrarily).

2°. Traveling-wave solution in implicit form:

$$\int \left[C + \frac{2}{A^2 + B^2} F(w)\right]^{-1/2} dw = Ax + By + D, \qquad F(w) = \int f(w)\,dw,$$

where A, B, C, and D are arbitrary constants.

3°. Solution with central symmetry about the point $(-C_1, -C_2)$:

$$w = w(\zeta), \qquad \zeta = \sqrt{(x + C_1)^2 + (y + C_2)^2},$$

where C_1 and C_2 are arbitrary constants and the function $w = w(\zeta)$ is determined by the ordinary differential equation $w''_{\zeta\zeta} + \zeta^{-1} w'_\zeta = f(w)$.

References

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