1. $\dfrac{\partial^2 w}{\partial x^2} + \dfrac{\partial}{\partial y}\left[(\alpha w + \beta)\dfrac{\partial w}{\partial y}\right] = 0.$

Stationary Khokhlov–Zabolotskaya equation. It arises in acoustics and nonlinear mechanics.

1°. Solutions:

$$w(x,y) = Ay - \tfrac{1}{2}A^2\alpha x^2 + C_1 x + C_2,$$
$$w(x,y) = (Ax+B)y - \frac{\alpha}{12A^2}(Ax+B)^4 + C_1 x + C_2,$$
$$w(x,y) = -\frac{1}{\alpha}\left(\frac{y+A}{x+B}\right)^2 + \frac{C_1}{x+B} + C_2(x+B)^2 - \frac{\beta}{\alpha},$$
$$w(x,y) = -\frac{1}{\alpha}\left[\beta + \lambda^2 \pm \sqrt{A(y+\lambda x)+B}\,\right],$$
$$w(x,y) = (Ax+B)\sqrt{C_1 y + C_2} - \frac{\beta}{\alpha},$$

where A, B, C_1, C_2, and λ are arbitrary constants.

2°. Generalized separable solution quadratic in y (generalizes the third solution of Item 1°):

$$w(x,y) = -\frac{1}{\alpha(x+A)^2}y^2 + \left[\frac{B_1}{(x+A)^2} + B_2(x+A)^3\right]y$$
$$+ \frac{C_1}{x+A} + C_2(x+A)^2 - \frac{\beta}{\alpha} - \frac{\alpha B_1^2}{4(x+A)^2} - \frac{1}{2}\alpha B_1 B_2 (x+A)^3 - \frac{1}{54}\alpha B_2^2 (x+A)^8,$$

where A, B_1, B_2, C_1, and C_2 are arbitrary constants.

3°. See also equation 3.3.3 with $f(w) = 1$ and $g(w) = \alpha w + \beta$.

References

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