2. $\dfrac{\partial^2 w}{\partial x^2} + \dfrac{\partial}{\partial y}\left(ae^{\beta w}\dfrac{\partial w}{\partial y}\right) = 0, \qquad a > 0.$

1°. Additive separable solutions:

$$w(x,y) = \frac{1}{\beta}\ln(Ay+B) + Cx + D,$$
$$w(x,y) = \frac{1}{\beta}\ln(-aA^2y^2+By+C) - \frac{2}{\beta}\ln(-aAx+D),$$
$$w(x,y) = \frac{1}{\beta}\ln(Ay^2+By+C) + \frac{1}{\beta}\ln\left[\frac{p^2}{aA\cosh^2(px+q)}\right],$$
$$w(x,y) = \frac{1}{\beta}\ln(Ay^2+By+C) + \frac{1}{\beta}\ln\left[\frac{p^2}{-aA\cos^2(px+q)}\right],$$
$$w(x,y) = \frac{1}{\beta}\ln(Ay^2+By+C) + \frac{1}{\beta}\ln\left[\frac{p^2}{-aA\sinh^2(px+q)}\right],$$

where A, B, C, D, p, and q are arbitrary constants.

2°. There are exact solutions of the following forms:

$$w(x,y) = F(r), \qquad r = k_1x + k_2y;$$
$$w(x,y) = G(z), \qquad z = y/x;$$
$$w(x,y) = H(\xi) - 2(k+1)\beta^{-1}\ln|x|, \qquad \xi = y|x|^k;$$
$$w(x,y) = U(\eta) - 2\beta^{-1}\ln|x|, \qquad \eta = y + k\ln|x|;$$
$$w(x,y) = V(\zeta) - 2\beta^{-1}x, \qquad \zeta = ye^x,$$

where k, k_1, and k_2 are arbitrary constants.

3°. For other solutions, see equation 3.3.3 with $f(w) = 1$ and $g(w) = ae^{\beta w}$.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Nonlinear Partial Differential Equations*, Chapman & Hall/CRC, Boca Raton, 2004.

