1. $\displaystyle a\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}=0.$

This is an ***equation of steady transonic gas flow***.

1°. Suppose $w(x,t)$ is a solution of the equation in question. Then the function

$$w_1=C_1^{-3}C_2^2 w(C_1x+C_3,\,C_2y+C_4)+C_5y+C_6,$$

where $C_1,\,\dots,\,C_6$ are arbitrary constants, is also a solution of the equation.

2°. Solutions:

$$w(x,y)=C_1xy+C_2x+C_3y+C_4,$$
$$w(x,y)=-\frac{(x+C_1)^3}{3a(y+C_2)^2}+C_3y+C_4,$$
$$w(x,y)=\frac{a^2C_1^3}{39}(y+A)^{13}+\frac{2}{3}aC_1^2(y+A)^8(x+B)+3C_1(y+A)^3(x+B)^2-\frac{(x+B)^3}{3a(y+A)^2},$$
$$w(x,y)=-aC_1y^2+C_2y+C_3\pm\frac{4}{3C_1}(C_1x+C_4)^{3/2},$$
$$w(x,y)=-aA^3y^2-\frac{B^2}{aA^2}x+C_1y+C_2\pm\frac{4}{3}(Ax+By+C_3)^{3/2},$$
$$w(x,y)=\frac{1}{3}(Ay+B)(2C_1x+C_2)^{3/2}-\frac{aC_1^3}{12A^2}(Ay+B)^4+C_3y+C_4,$$
$$w(x,y)=-\frac{9aA^2}{y+C_1}+4A\left(\frac{x+C_2}{y+C_1}\right)^{3/2}-\frac{(x+C_2)^3}{3a(y+C_1)^2}+C_3y+C_4,$$
$$w(x,y)=-\frac{3}{7}aA^2(y+C_1)^7+4A(x+C_2)^{3/2}(y+C_1)^{5/2}-\frac{(x+C_2)^3}{3a(y+C_1)^2}+C_3y+C_4,$$

where A, B, $C_1,\,\dots,\,C_4$ are arbitrary constants. (the first solution is degenerate).

3°. There are solutions of the following forms:

$w(x,y)=y^{-3k-2}U(z),\quad z=xy^k$ — self-similar solution, k is any;

$w(x,y)=\varphi_1(y)+\varphi_2(y)x^{3/2}+\varphi_3(y)x^3$ — generalized separable solution;

$w(x,y)=\psi_1(y)+\psi_2(y)x+\psi_3(y)x^2+\psi_4(y)x^3$ — generalized separable solution;

$w(x,y)=\psi_1(y)\varphi(x)+\psi_2(y)$ — generalized separable solution.

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