2. $\dfrac{\partial^2 w}{\partial y^2} + \dfrac{a}{y}\dfrac{\partial w}{\partial y} + b\dfrac{\partial w}{\partial x}\dfrac{\partial^2 w}{\partial x^2} = 0.$

This is an ***generalized equation of steady transonic gas flow***.

1°. Suppose $w(x,t)$ is a solution of the equation in question. Then the function

$$w_1 = C_1^{-3}C_2^2 w(C_1 x + C_3, C_2 y) + C_4 y^{1-a} + C_5,$$

where $C_1, \dots, C_5$ are arbitrary constants, is also a solution of the equation.

2°. Additive separable solution:

$$w(x,y) = -\frac{bC_1}{4(a+1)}y^2 + C_2 y^{1-a} + C_3 \pm \frac{2}{3C_1}(C_1 x + C_4)^{3/2},$$

where $C_1, \dots, C_4$ are arbitrary constants.

3°. Generalized separable solutions:

$$w(x,y) = -\frac{9A^2 b}{16(n+1)(2n+1+a)}y^{2n+2} + Ay^n(x+C)^{3/2} + \frac{a-3}{9b}\frac{(x+C)^3}{y^2},$$

where A and C are arbitrary constants, and the $n = n_{1,2}$ are roots of the quadratic equation $n^2 + (a-1)n + \frac{5}{4}(a-3) = 0$.

4°. Generalized separable solution:

$$w(x,y) = (Ay^{1-a} + B)(2C_1 x + C_2)^{3/2} + 9bC_1^3\theta(y),$$
$$\theta(y) = -\frac{B^2}{2(a+1)}y^2 - \frac{AB}{3-a}y^{3-a} - \frac{A^2}{2(2-a)(3-a)}y^{4-2a} + C_3 y^{1-a} + C_4,$$

where A, B, C_1, C_2, C_3, and C_4 are arbitrary constants.

5°. There are solutions of the following forms:

$w(x,y) = y^{-3k-2}U(z), \quad z = xy^k$	self-similar solution, k is any;
$w(x,y) = \varphi_1(y) + \varphi_2(y)x^{3/2} + \varphi_3(y)x^3$	generalized separable solution;
$w(x,y) = \psi_1(y) + \psi_2(y)x + \psi_3(y)x^2 + \psi_4(y)x^3$	generalized separable solution;
$w(x,y) = \psi_1(y)\varphi(x) + \psi_2(y)$	generalized separable solution.

References

Titov, S. S., A method of finite-dimensional rings for solving nonlinear equations of mathematical physics [in Russian], In: *Aerodynamics* (Editor T. P. Ivanova), Saratov Univ., Saratov, pp. 104–110, 1988.

Svirshchevskii, S. R., Lie–Bäcklund symmetries of linear ODEs and generalized separation of variables in nonlinear equations, *Phys. Lett. A*, Vol. 199, pp. 344–348, 1995.

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Nonlinear Partial Differential Equations* , Chapman & Hall/CRC, Boca Raton, 2004.

