



Exact Solutions > Nonlinear Partial Differential Equations > Other Second-Order Partial Differential Equations > Homogeneous Monge–Ampère Equation (Monge–Ampère Equation)

$$1. \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} = 0.$$

Homogeneous Monge–Ampère equation (Monge–Ampère Equation).

1°. General solution in parametric form:

$$\begin{aligned} w &= tx + \varphi(t)y + \psi(t), \\ x + \varphi'(t)y + \psi'(t) &= 0, \end{aligned}$$

where t is the parameter, and $\varphi = \varphi(t)$ and $\psi = \psi(t)$ are arbitrary functions.

2°. Solutions involving one arbitrary function:

$$\begin{aligned} w(x, y) &= \varphi(C_1x + C_2y) + C_3x + C_4y + C_5, \\ w(x, y) &= (C_1x + C_2y)\varphi\left(\frac{y}{x}\right) + C_3x + C_4y + C_5, \\ w(x, y) &= (C_1x + C_2y + C_3)\varphi\left(\frac{C_4x + C_5y + C_6}{C_1x + C_2y + C_3}\right) + C_7x + C_8y + C_9, \end{aligned}$$

where C_1, \dots, C_9 are arbitrary constants and $\varphi = \varphi(z)$ is an arbitrary function.

See also:

- [nonhomogeneous Monge–Ampère equation \(special case\)](#),
- [nonhomogeneous Monge–Ampère equation](#).

References

- Goursat, E.**, *A Course of Mathematical Analysis. Vol 3. Part 1* [Russian translation], Gostekhizdat, Moscow, 1933.
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- Ibragimov, N. H.** (Editor), *CRC Handbook of Lie Group Analysis of Differential Equations, Vol. 1, Symmetries, Exact Solutions and Conservation Laws*, CRC Press, Boca Raton, 1994.
- Polyanin, A. D. and Zaitsev, V. F.**, *Handbook of Nonlinear Partial Differential Equations*, Chapman & Hall/CRC, Boca Raton, 2004.

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