2. $\displaystyle \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} = A.$

Nonhomogeneous Monge–Ampere equation (Monge–Ampère equation).

1°. General solution in parametric form for $A = a^2 > 0$:

$$x = \frac{\beta - \lambda}{2a}, \quad y = \frac{\psi'(\lambda) - \varphi'(\beta)}{2a}, \quad w = \frac{(\beta+\lambda)[\psi'(\lambda) - \varphi'(\beta)] + 2\varphi(\beta) - 2\psi(\lambda)}{4a},$$

where β and λ are the parameters, $\varphi = \varphi(\beta)$ and $\psi = \psi(\lambda)$ are arbitrary functions.

2°. Solutions:

$$w(x,y) = \pm\frac{\sqrt{A}}{C_2}x(C_1x + C_2y) + \varphi(C_1x + C_2y) + C_3x + C_4y,$$

$$w(x,y) = C_1y^2 + C_2xy + \frac{1}{4C_1}(C_2^2 - A)x^2 + C_3y + C_4x + C_5,$$

$$w(x,y) = \frac{1}{x + C_1}\left(C_2y^2 + C_3y + \frac{C_3^2}{4C_2}\right) - \frac{A}{12C_2}(x^3 + 3C_1x^2) + C_4y + C_5x + C_6,$$

$$w(x,y) = \pm\frac{2\sqrt{A}}{3C_1C_2}(C_1x - C_2^2y^2 + C_3)^{3/2} + C_4x + C_5y + C_6,$$

where $C_1, \ldots, C_6$ are arbitrary constants and $\varphi = \varphi(z)$ is an arbitrary function.

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