



Exact Solutions > Nonlinear Partial Differential Equations > Other Second-Order Partial Differential Equations > Nonhomogeneous Monge–Ampère Equation (Monge–Ampère Equation)

$$3. \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} = F(x, y).$$

Nonhomogeneous Monge–Ampère equation (Monge–Ampère equation).

1°. Particular cases:

- for $F(x, y) = 0$ see [homogeneous Monge–Ampère equation](#) ;
- for $F(x, y) = A$ see [nonhomogeneous Monge–Ampère equation \(special case\)](#) .

2°. Table 1 presents all other nonhomogeneous Monge–Ampère equations whose exact solutions are outlined in *Handbook of Nonlinear Partial Differential Equations* by Polyanin & Zaitsev. The number of the equation sought is indicated in the last column; $f(x)$, $g(x)$, and $h(x)$ are arbitrary functions; $a, b, c, k, s, \alpha, \beta$, and λ are arbitrary constants.

TABLE 1

Nonhomogeneous Monge–Ampère equations of the form $w_{xy}^2 - w_{xx}w_{yy} = F(x, y)$ whose exact solutions are outlined in *Handbook of Nonlinear Partial Differential Equations*

No.	Function $F(x, y)$	Equation
1	$f(x)$	7.2.2.3
2	$f(x)y$	7.2.2.4
3	$f(x)y^2$	7.2.2.5
4	$f(x)y^2 + g(x)y + h(x)$	7.2.2.6
5	$f(x)y^k$	7.2.2.7
6	$f(x)y^{2k+2} + g(x)y^k$	7.2.2.8
7	$f(x)e^{\lambda y}$	7.2.2.9
8	$f(x)e^{2\lambda y} + g(x)e^{\lambda y}$	7.2.2.10
9	$f(x)g(y) + a^2$	7.2.2.11
10	$f(ax + by)$	7.2.2.12
11	$x^k f(ax + by)$	7.2.2.13
12	$x^{2k+2} f(ax + by) + x^k g(ax + by)$	7.2.2.14
13	$e^{\lambda x} f(ax + by)$	7.2.2.15
14	$e^{2\lambda x} f(ax + by) + e^{\lambda x} g(ax + by)$	7.2.2.16
15	$x^{-4} f(y/x)$	7.2.2.17
16	$x^\alpha f(x^\beta y)$	7.2.2.18

TABLE 1 (Continued)
 Nonhomogeneous Monge–Ampere equations of the form $w_{xy}^2 - w_{xx}w_{yy} = F(x, y)$ whose exact solutions are outlined in *Handbook of Nonlinear Partial Differential Equations*

No.	Function $F(x, y)$	Equation
17	$f(ax - by^2)$	7.2.2.19
18	$f(ax^2 + bxy + cy^2)$	7.2.2.20
19	$f(ax^2 + bxy + cy^2 + kx + sy)$	7.2.2.21
20	$e^{\alpha x} f(e^{\beta x} y)$	7.2.2.22
20	$e^{ky/x} f(x)$	7.2.2.23
21	$x^\alpha f(x^\beta y)$	7.2.2.24
22	$y^{-4} \exp(\alpha y^{-1}) f(xy^{-1} + \beta y^{-2})$	7.2.2.25

References

- Rozhdestvenskii, B. L. and Yanenko, N. N.**, *Systems of Quasilinear Equations and Their Applications to Gas Dynamics*, Amer. Math. Society, Providence, 1983.
- Khabirov, S. V.**, Nonisentropic one-dimensional gas motions obtained with the help of the contact group of the nonhomogeneous Monge–Ampere equation, [in Russian], *Mat. Sbornik*, Vol. 181, No. 12, pp. 1607–1622, 1990.
- Ibragimov, N. H.** (Editor), *CRC Handbook of Lie Group Analysis of Differential Equations, Vol. 1, Symmetries, Exact Solutions and Conservation Laws*, CRC Press, Boca Raton, 1994.
- Polyanin, A. D. and Zaitsev, V. F.**, *Handbook of Nonlinear Partial Differential Equations*, Chapman & Hall/CRC, Boca Raton, 2004.