3. \( \frac{\partial w}{\partial t} + \frac{\partial^3 w}{\partial x^3} + 6\sigma w^2 \frac{\partial w}{\partial x} = 0. \)

**Modified Korteweg–de Vries equation.**

1°. One-soliton solution for \( \sigma = 1 \):

\[
\begin{align*}
    w(x, t) &= a + \frac{k^2}{\sqrt{4a^2 + k^2}} \cosh z + 2a, \\
    z &= kx - (6a^2k + k^3)t + b,
\end{align*}
\]

where \( a, b, \) and \( k \) are arbitrary constants.

2°. Two-soliton solution for \( \sigma = 1 \):

\[
\begin{align*}
    w(x, t) &= 2a_1e^{\theta_1} + a_2e^{\theta_2} + \frac{Aa_2^2e^{2\theta_1} + Aa_1e^{2\theta_1 + 2\theta_2}}{1 + e^{2\theta_1} + e^{2\theta_2} + 2(1 - A)e^{\theta_1 + \theta_2} + 2Ae^{2\theta_1 + \theta_2}}, \\
    \theta_1 &= a_1x - a_1^3t + b_1, \\
    \theta_2 &= a_2x - a_2^3t + b_2, \\
    A &= \left( \frac{a_1 - a_2}{a_1 + a_2} \right)^2,
\end{align*}
\]

where \( a_1, a_2, b_1, \) and \( b_2 \) are arbitrary constants.

3°. Rational solutions (algebraic solitons) for \( \sigma = 1 \):

\[
\begin{align*}
    w(x, t) &= a - \frac{4a}{4a^2z^2 + 1}, \\
    w(x, t) &= a - \frac{12a(z^4 + \frac{3}{2}a^2z^2 - \frac{3}{38}a^4 - 24tz)}{4a^2(z^3 + 12t - \frac{1}{4}a^2z^2)^2 + 3(z^2 + \frac{1}{4}a^2)^2},
\end{align*}
\]

where \( a \) is an arbitrary constant.

4°. There is a self-similar solution of the form \( w = \ell^{-1/3}U(z) \), where \( z = \ell^{-1/3}x \).

5°. The modified Korteweg–de Vries equation is solved by the inverse scattering method.

**References**


