1. First-Order Ordinary Differential Equations

1. $y'_x = f(y).$ *Autonomous equation.*

2. $y'_x = f(x)g(y).$ *Separable equation.*

3. $g(x)y'_x = f_1(x)y + f_0(x).$ *Linear equation.*

4. $g(x)y'_x = f_1(x)y + f_n(x)y^n.$ *Bernoulli equation.*

5. $y'_x = f(y/x).$ *Homogeneous equation.*

6. $y'_x = ay^2 + bx^n.$ *Special Riccati equation.*

7. $y'_x = y^2 + f(x)y - a^2 - af(x).$ *Riccati equation, special case 1.*

8. $y'_x = f(x)y^2 + ay - ab - b^2f(x).$ *Riccati equation, special case 2.*

9. $y'_x = y^2 + xf(x)y + f(x).$ *Riccati equation, special case 3.*

10. $y'_x = f(x)y^2 - ax^nf(x)y + anx^{n-1}.$ *Riccati equation, special case 4.*

11. $y'_x = f(x)y^2 + anx^{n-1} - a^2x^{2n}f(x).$ *Riccati equation, special case 5.*

12. $y'_x = -(n+1)x^ny^2 + x^{n+1}f(x)y - f(x).$ *Riccati equation, special case 6.*

13. $xy'_x = f(x)y^2 + ny + ax^{2n}f(x).$ *Riccati equation, special case 7.*

14. $xy'_x = x^{2n}f(x)y^2 + [ax^nf(x) - n]y + bf(x).$ *Riccati equation, special case 8.*

15. $y'_x = f(x)y^2 + g(x)y - a^2f(x) - ag(x).$ *Riccati equation, special case 9.*

16. $y'_x = f(x)y^2 + g(x)y + anx^{n-1} - a^2x^{2n}f(x) - ax^ng(x).$
  Riccati equation, special case 10.

17. $y'_x = ae^{\lambda x}y^2 + ae^{\lambda x}f(x)y + \lambda f(x).$ *Riccati equation, special case 11.*

18. $y'_x = f(x)y^2 - ae^{\lambda x}f(x)y + a\lambda e^{\lambda x}.$ *Riccati equation, special case 12.*

19. $y'_x = f(x)y^2 + a\lambda e^{\lambda x} - a^2e^{2\lambda x}f(x).$ *Riccati equation, special case 13.*

20. $y'_x = f(x)y^2 + \lambda y + ae^{2\lambda x}f(x).$ *Riccati equation, special case 14.*

21. $y'_x = y^2 - f^2(x) + f'_x(x).$ *Riccati equation, special case 15.*

22. $y'_x = f(x)y^2 - f(x)g(x)y + g'_x(x).$ *Riccati equation, special case 16.*

23. $y'_x = f(x)y^2 + g(x)y + h(x).$ *General Riccati equation.*

24. $yy'_x = y + f(x).$ *Abel equation of the second kind in the canonical form.*

25. $yy'_x = f(x)y + g(x)$.   *Abel equation of the second kind.*

26. $yy'_x = f(x)y^2 + g(x)y + h(x)$.   *Abel equation of the second kind.*

27. $y'_x = f(ax + by + c)$.

28. $y'_x = f(y + ax^n + b) - anx^{n-1}$.

29. $y'_x = \dfrac{y}{x} f(x^n y^m)$.   *Generalized homogeneous equation.*

30. $y'_x = -\dfrac{n}{m}\dfrac{y}{x} + y^k f(x)g(x^n y^m)$.

31. $y'_x = f\left(\dfrac{ax + by + c}{\alpha x + \beta y + \gamma}\right)$.

32. $y'_x = x^{n-1}y^{1-m} f(ax^n + by^m)$.

33. $[x^n f(y) + xg(y)]y'_x = h(y)$.

34. $x[f(x^n y^m) + mx^k g(x^n y^m)]y'_x = y[h(x^n y^m) - nx^k g(x^n y^m)]$.

35. $x[f(x^n y^m) + my^k g(x^n y^m)]y'_x = y[h(x^n y^m) - ny^k g(x^n y^m)]$.

36. $x[sf(x^n y^m) - mg(x^k y^s)]y'_x = y[ng(x^k y^s) - kf(x^n y^m)]$.

37. $[f(y) + amx^n y^{m-1}]y'_x + g(x) + anx^{n-1}y^m = 0$.

38. $y'_x = e^{-\lambda x} f(e^{\lambda x}y)$.

39. $y'_x = e^{\lambda y} f(e^{\lambda y}x)$.

40. $y'_x = yf(e^{\alpha x}y^m)$.

41. $y'_x = \dfrac{1}{x} f(x^n e^{\alpha y})$.

42. $y'_x = f(x)e^{\lambda y} + g(x)$.

43. $y'_x = -\dfrac{n}{x} + f(x)g(x^n e^y)$.

44. $y'_x = -\dfrac{\alpha}{m}y + y^k f(x)g(e^{\alpha x}y^m)$.

45. $y'_x = e^{\alpha x - \beta y} f(ae^{\alpha x} + be^{\beta y})$.

46. $[e^{\alpha x} f(y) + a\beta]y'_x + e^{\beta y} g(x) + a\alpha = 0$.

47. $x[f(x^n e^{\alpha y}) + \alpha yg(x^n e^{\alpha y})]y'_x = h(x^n e^{\alpha y}) - nyg(x^n e^{\alpha y})$.

48. $[f(e^{\alpha x}y^m) + mxg(e^{\alpha x}y^m)]y'_x = y[h(e^{\alpha x}y^m) - \alpha xg(e^{\alpha x}y^m)]$.

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