



## 3. Second-Order Nonlinear Ordinary Differential Equations

### 3.1. Ordinary Differential Equations of the Form $y''_{xx} = f(x, y)$

1.  $y''_{xx} = f(y)$ . *Autonomous equation.*
2.  $y''_{xx} = Ax^n y^m$ . *Emden–Fowler equation.*
3.  $y''_{xx} + f(x)y = ay^{-3}$ . *Ermakov (Yermakov) equation.*
4.  $y''_{xx} = f(ay + bx + c)$ .
5.  $y''_{xx} = f(y + ax^2 + bx + c)$ .
6.  $y''_{xx} = x^{-1}f(yx^{-1})$ . *Homogeneous equation.*
7.  $y''_{xx} = x^{-3}f(yx^{-1})$ .
8.  $y''_{xx} = x^{-3/2}f(yx^{-1/2})$ .
9.  $y''_{xx} = x^{k-2}f(x^{-k}y)$ . *Generalized homogeneous equation.*
10.  $y''_{xx} = yx^{-2}f(x^n y^m)$ . *Generalized homogeneous equation.*
11.  $y''_{xx} = y^{-3}f\left(\frac{y}{\sqrt{ax^2 + bx + c}}\right)$ .
12.  $y''_{xx} = e^{-ax}f(e^{ax}y)$ .
13.  $y''_{xx} = yf(e^{ax}y^m)$ .
14.  $y''_{xx} = x^{-2}f(x^n e^{ay})$ .
15.  $y''_{xx} = \frac{\psi''_{xx}}{\psi}y + \psi^{-3}f\left(\frac{y}{\psi}\right)$ ,  $\psi = \psi(x)$ .

### 3.2. Ordinary Differential Equations of the Form

$$f(x, y)y''_{xx} = g(x, y, y'_x)$$

16.  $y''_{xx} - y'_x = f(y)$ . *Autonomous equation.*
17.  $y''_{xx} + f(y)y'_x + g(y) = 0$ . *Lienard equation.*
18.  $y''_{xx} + [ay + f(x)]y'_x + f'_x(x)y = 0$ .
19.  $y''_{xx} + [2ay + f(x)]y'_x + af(x)y^2 = g(x)$ .
20.  $y''_{xx} = ay'_x + e^{2ax}f(y)$ .

21.  $y''_{xx} = f(y)y'_x.$
22.  $y''_{xx} = [e^{\alpha x} f(y) + \alpha] y'_x.$
23.  $xy''_{xx} = ny'_x + x^{2n+1} f(y).$
24.  $xy''_{xx} = f(y)y'_x.$
25.  $xy''_{xx} = [x^k f(y) + k - 1] y'_x.$
26.  $x^2 y''_{xx} + xy'_x = f(y).$
27.  $(ax^2 + b)y''_{xx} + axy'_x + f(y) = 0.$
28.  $y''_{xx} = f(y)y'_x + g(x).$
29.  $xy''_{xx} + (n + 1)y'_x = x^{n-1} f(yx^n).$
30.  $gy''_{xx} + \frac{1}{2}g'_x y'_x = f(y), \quad g = g(x).$
31.  $y''_{xx} = -ay'_x + e^{ax} f(ye^{ax}).$
32.  $xy''_{xx} = f(x^n e^{ay})y'_x.$
33.  $x^2 y''_{xx} + xy'_x = f(x^n e^{ay}).$
34.  $yy''_{xx} + (y'_x)^2 + f(x)yy'_x + g(x) = 0.$
35.  $yy''_{xx} - (y'_x)^2 + f(x)yy'_x + g(x)y^2 = 0.$
36.  $yy''_{xx} - n(y'_x)^2 + f(x)y^2 + ay^{4n-2} = 0.$
37.  $yy''_{xx} - n(y'_x)^2 + f(x)y^2 + g(x)y^{n+1} = 0.$
38.  $yy''_{xx} + a(y'_x)^2 + f(x)yy'_x + g(x)y^2 = 0.$
39.  $yy''_{xx} = f(x)(y'_x)^2.$
40.  $y''_{xx} - a(y'_x)^2 + f(x)e^{ay} + g(x) = 0.$
41.  $y''_{xx} - a(y'_x)^2 + be^{4ay} + f(x) = 0.$
42.  $y''_{xx} + a(y'_x)^2 - \frac{1}{2}y'_x = e^x f(y).$
43.  $y''_{xx} + \alpha(y'_x)^2 = [e^{\beta x} f(y) + \beta] y'_x.$
44.  $y''_{xx} + f(y)(y'_x)^2 + g(y) = 0.$
45.  $y''_{xx} + f(y)(y'_x)^2 - \frac{1}{2}y'_x = e^x g(y).$
46.  $y''_{xx} = xf(y)(y'_x)^3.$
47.  $y''_{xx} = f(y)(y'_x)^2 + g(x)y'_x.$

$$48. \quad y''_{xx} = f(x)g(xy'_x - y).$$

$$49. \quad y''_{xx} = \frac{y}{x^2} f\left(\frac{xy'_x}{y}\right).$$

$$50. \quad gy''_{xx} + \frac{1}{2}g'_x y'_x = f(y)h(y'_x \sqrt{g}), \quad g = g(x).$$

$$51. \quad y''_{xx} = f(y'^2 + ay).$$

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The EqWorld website presents extensive information on solutions to various classes of ordinary differential equations, partial differential equations, integral equations, functional equations, and other mathematical equations.

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<http://eqworld.ipmnet.ru/en/solutions/ode/ode-toc3.pdf>