19. $y'_x = f(x)y^2 + a\lambda e^{\lambda x} - a^2 e^{2\lambda x} f(x)$.

Riccati equation, special case 13.

Particular solution: $y_0 = ae^{\lambda x}$.

The general solution can be written as:

$$y = ae^{\lambda x} + \Phi(x)\left[C - \int f(x)\Phi(x)\,dx\right]^{-1}, \quad \text{where} \quad \Phi(x) = \exp\left[2a\int e^{\lambda x} f(x)\,dx\right],$$

C is an arbitrary constant.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition* , Chapman & Hall/CRC, Boca Raton, 2003.

