21. $y'_x = y^2 - f^2(x) + f'_x(x)$.

Riccati equation, special case 15.

Particular solution: $y_0 = f(x)$.

The general solution can be written as:

$$y = f(x) + \Phi(x)\left[C - \int \Phi(x)\,dx\right]^{-1}, \quad \text{where} \quad \Phi(x) = \exp\left[2\int f(x)\,dx\right],$$

C is an arbitrary constant.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition* , Chapman & Hall/CRC, Boca Raton, 2003.

