2. $y''_{xx} - ax^n y = 0.$

1°. Solution for $n = -2$:

$$y = C_1|x|^{k_1} + C_2|x|^{k_2},$$

where C_1 and C_2 are arbitrary constants, k_1 and k_2 are roots of the quadratic equation $k^2 - k - a = 0$.

2°. Assume $2/(n+2) = 2m+1$, where m is an integer. Then the solution is:

$$y = \begin{cases} x(x^{1-2q}D)^{m+1}\left[C_1\exp\left(\frac{\sqrt{a}}{q}x^q\right) + C_2\exp\left(-\frac{\sqrt{a}}{q}x^q\right)\right] & \text{if } m \ge 0, \\ (x^{1-2q}D)^{-m}\left[C_1\exp\left(\frac{\sqrt{a}}{q}x^q\right) + C_2\exp\left(-\frac{\sqrt{a}}{q}x^q\right)\right] & \text{if } m < 0, \end{cases}$$

where $D = \dfrac{d}{dx}$, $q = \dfrac{n+2}{2} = \dfrac{1}{2m+1}$.

3°. For any n, the solution is expressed in terms of the Bessel functions and modified Bessel functions:

$$y = \begin{cases} C_1\sqrt{x}\,J_{\frac{1}{2q}}\left(\frac{\sqrt{-a}}{q}x^q\right) + C_2\sqrt{x}\,Y_{\frac{1}{2q}}\left(\frac{\sqrt{-a}}{q}x^q\right) & \text{if } a < 0, \\ C_1\sqrt{x}\,I_{\frac{1}{2q}}\left(\frac{\sqrt{a}}{q}x^q\right) + C_2\sqrt{x}\,K_{\frac{1}{2q}}\left(\frac{\sqrt{a}}{q}x^q\right) & \text{if } a > 0, \end{cases}$$

where $q = \frac{1}{2}(n+2)$.

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