14. $x^2 y''_{xx} + x y'_x - (x^2 + \nu^2) y = 0.$

Modified Bessel equation. It can be reduced to the Bessel equation by means of the substitution $x = i\bar{x}$, where $i^2 = -1$.

Solution:

$$y = C_1 I_\nu(x) + C_2 K_\nu(x),$$

where C_1 and C_2 are arbitrary constants, $I_\nu(x)$ and $K_\nu(x)$ are the modified Bessel functions of the first and second kind:

$$I_\nu(x) = \sum_{k=0}^{\infty} \frac{(x/2)^{2k+\nu}}{k!\,\Gamma(\nu+k+1)}, \qquad K_\nu(x) = \frac{\pi}{2}\,\frac{I_{-\nu}(x) - I_\nu(x)}{\sin \pi\nu}.$$

References

Bateman, H. and Erdélyi, A., *Higher Transcendental Functions*, Vol. 2, McGraw-Hill, New York, 1953.

Abramowitz, M. and Stegun, I. A. (Editors), *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables*, National Bureau of Standards Applied Mathematics, Washington, 1964.

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition* , Chapman & Hall/CRC, Boca Raton, 2003.

