19. $(1-x^2)y''_{xx} - 2xy'_x + \nu(\nu+1)y = 0.$

Legendre equation, special case 2; ν is an arbitrary number. The case $\nu = n$ where n is a nonnegative integer is considered in 2.18 .

The substitution $z = x^2$ leads to the hypergeometric equation. Therefore, with $|x| < 1$ the solution can be written as:

$$y = C_1 F\left(-\frac{\nu}{2}, \frac{1+\nu}{2}, \frac{1}{2}; x\right) + C_2 x F\left(\frac{1-\nu}{2}, 1+\frac{\nu}{2}, \frac{3}{2}; x\right),$$

where $F(\alpha, \beta, \gamma; x)$ is the hypergeometric series (see equation 2.22).

References

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