



Exact Solutions > Ordinary Differential Equations > Second-Order Linear Ordinary Differential Equations > Legendre Equation, Special Case 2

19. $(1 - x^2)y''_{xx} - 2xy'_x + \nu(\nu + 1)y = 0.$

Legendre equation, special case 2; ν is an arbitrary number. The case $\nu = n$ where n is a nonnegative integer is considered in [2.18](#).

The substitution $z = x^2$ leads to the hypergeometric equation. Therefore, with $|x| < 1$ the solution can be written as:

$$y = C_1 F\left(-\frac{\nu}{2}, \frac{1+\nu}{2}, \frac{1}{2}; x\right) + C_2 x F\left(\frac{1-\nu}{2}, 1 + \frac{\nu}{2}, \frac{3}{2}; x\right),$$

where $F(\alpha, \beta, \gamma; x)$ is the hypergeometric series (see [equation 2.22](#)).

References

Abramowitz, M. and Stegun, I. A. (Editors), *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables*, National Bureau of Standards Applied Mathematics, Washington, 1964.

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.

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