23. $(1-x^2)^2 y''_{xx} - 2x(1-x^2)y'_x + [\nu(\nu+1)(1-x^2) - \mu^2]y = 0.$

Legendre equation, ν and μ are arbitrary parameters.

The transformation $x = 1-2\xi$, $y = |x^2-1|^{\mu/2} w$ leads to the hypergeometric equation 2.22 :

$$\xi(\xi-1)w''_{\xi\xi} + (\mu+1)(1-2\xi)w'_\xi + (\nu-\mu)(\nu+\mu+1)w = 0$$

with parameters $\alpha = \mu-\nu$, $\beta = \mu+\nu+1$, $\gamma = \mu+1$.

In particular, the original equation is integrable by quadrature if $\nu = \mu$ or $\nu = -\mu-1$.

See also special cases of the Legendre equation:

- Legendre equation, special case 1 ,
- Legendre equation, special case 2 .

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