



Exact Solutions > Ordinary Differential Equations > Second-Order Linear Ordinary Differential Equations > Modified Mathieu Equation

**33.  $y''_{xx} - (a - 2q \cosh 2x)y = 0.$**

**Modified Mathieu equation.** The substitution  $x = i\xi$  leads to the [Mathieu equation 2.34](#) :

$$y''_{\xi\xi} + (a - 2q \cos 2\xi)y = 0.$$

For eigenvalues  $a = a_n(q)$  and  $a = b_n(q)$ , the corresponding solutions of the modified Mathieu equation are:

$$\begin{aligned} \text{Ce}_{2n+p}(x, q) &= \text{ce}_{2n+p}(ix, q) = \sum_{k=0}^{\infty} A_{2k+p}^{2n+p} \cosh[(2k+p)x], \\ \text{Se}_{2n+p}(x, q) &= -i \text{se}_{2n+p}(ix, q) = \sum_{k=0}^{\infty} B_{2k+p}^{2n+p} \sinh[(2k+p)x], \end{aligned}$$

where  $p$  can be either 0 or 1, and the coefficients  $A_{2k+p}^{2n+p}$  and  $B_{2k+p}^{2n+p}$  are specified in 2.34.

### References

- McLachlan, N. W.**, *Theory and Application of Mathieu Functions*, Clarendon Press, Oxford, 1947.
- Bateman, H. and Erdélyi, A.**, *Higher Transcendental Functions*, Vol. 3, McGraw-Hill, New York, 1955.
- Abramowitz, M. and Stegun, I. A. (Editors)**, *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables*, National Bureau of Standards Applied Mathematics, Washington, 1964.
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