



39. $xy''_{xx} + [(ax + 1)f + ax - 1]y'_x + a^2xfy = 0, \quad f = f(x).$

Particular solution: $y_0 = (ax + 1)e^{-ax}$.

Solution:

$$y = y_0 \left(C_1 + C_2 \int \frac{e^{-F}}{y_0^2} dx \right), \quad \text{where } F = ax - \ln|x| + \int (a + x^{-1})f dx,$$

C_1 and C_2 are arbitrary constants.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.