46. $y''_{xx} + fy'_x + (fg - g^2 + g'_x)y = 0, \qquad f = f(x).$

Particular solution: $y_0 = \exp\left(-\int g\,dx\right)$.

Solution:

$$y = y_0\left(C_1 + C_2\int \frac{e^{-F}}{y_0^2}\,dx\right), \qquad \text{where} \quad F = \int f\,dx,$$

C_1 and C_2 are arbitrary constants.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.

