



2. $y''_{xx} = Ax^n y^m$.

Emden–Fowler equation.

1°. With $m \neq 1$, the Emden–Fowler equation has a particular solution:

$$y = \lambda x^{\frac{n+2}{1-m}}, \quad \text{where } \lambda = \left[\frac{(n+2)(n+m+1)}{A(m-1)^2} \right]^{\frac{1}{m-1}}.$$

2°. The transformation $z = x^{n+2} y^{m-1}$, $w = xy'_x/y$ leads to a first-order (Abel) equation:

$$z[(m-1)w + n + 2]w'_z = -w^2 + w + Az.$$

3°. The transformation $y = w/t$, $x = 1/t$ leads to the Emden–Fowler equation with the independent variable raised to a different power: $w''_{tt} = At^{-n-m-3}w^m$.

4°. Table 1 presents all solvable Emden–Fowler equations whose solutions are outlined in *Handbook of Exact Solutions for Ordinary Differential Equations* by Polyanin & Zaitsev. The one-parameter families (in the space of the parameters n and m) and isolated points are presented in a consecutive fashion. Equations are arranged in order of increasing m and increasing n (for identical m). The number of the equation sought is indicated in the last column.

TABLE 1
Solvable cases of the Emden–Fowler equation $y''_{xx} = Ax^n y^m$

No	m	n	Equation	No	m	n	Equation
<i>One-parameter families</i>				13	-5/3	-5/6	2.3.1.23
1	arbitrary	0	2.3.1.2	14	-5/3	-1/2	2.3.1.24
2	arbitrary	-m - 3	2.3.1.3	15	-5/3	1	2.3.1.7
3	arbitrary	-1/2(m + 3)	2.3.1.4	16	-5/3	2	2.3.1.9
4	0	arbitrary	2.3.1.1	17	-7/5	-13/5	2.3.1.14
5	1	arbitrary	2.3.1.5	18	-7/5	1	2.3.1.13
<i>Isolated points</i>				19	-1/2	-7/2	2.3.1.12
6	-7	1	2.3.1.15	20	-1/2	-5/2	2.3.1.6
7	-7	3	2.3.1.16	21	-1/2	-2	2.3.1.26
8	-5/2	-1/2	2.3.1.22	22	-1/2	-4/3	2.3.1.17
9	-2	-2	2.3.1.28	23	-1/2	-7/6	2.3.1.18
10	-2	1	2.3.1.27	24	-1/2	-1/2	2.3.1.25
11	-5/3	-10/3	2.3.1.10	25	-1/2	1	2.3.1.11
12	-5/3	-7/3	2.3.1.8	26	2	-5	2.3.1.19
				27	2	-20/7	2.3.1.21
				28	2	-15/7	2.3.1.20

References

Kamke, E., *Differentialgleichungen: Lösungsmethoden und Lösungen, I, Gewöhnliche Differentialgleichungen*, B. G. Teubner, Leipzig, 1977.

Zaitsev, V. F. and Polyanin, A. D., *Discrete-Group Methods for Integrating Equations of Nonlinear Mechanics*, CRC Press, Boca Raton, 1994.

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.

Emden–Fowler Equation

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<http://eqworld.ipmnet.ru/en/solutions/ode/ode0302.pdf>