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3. $y''_{xx} + f(x)y = ay^{-3}$.

Ermakov (Yermakov) equation.

1°. Let $w = w(x)$ be a nontrivial solution of the second-order linear equation

$$w''_{xx} + f(x)w = 0.$$

The transformation

$$\xi = \int \frac{dx}{w^2}, \quad z = \frac{y}{w}$$

leads the Ermakov equation to an autonomous equation of the form 3.1: $z''_{\xi\xi} = az^{-3}$.

2°. Solution:

$$C_1 y^2 = aw^2 + w^2 \left(C_2 + C_1 \int \frac{dx}{w^2} \right)^2,$$

where C_1 and C_2 are arbitrary constants.

References

- Ermakov, V. P.**, Second-order differential equations. Integrability conditions in closed form [in Russian], *Universitetskie Izvestiya*, Kiev, No. 9, pp. 1–25, 1880.
- Berkovich, L. M.**, *Factorization and Transformations of Ordinary Differential Equations* [in Russian], Regulyarnaya i Khaoticheskaya Dinamika, Moscow, 2002.
- Polyanin, A. D. and Zaitsev, V. F.**, *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.

Ermakov equation