8. $y''_{xx} = x^{-3/2} f(yx^{-1/2})$.

Having set $w = yx^{-1/2}$, we obtain

$$\frac{d}{dx}(xw'_x)^2 = \tfrac{1}{2}ww'_x + 2f(w)w'_x.$$

Integrating the latter equation, we arrive at a separable equation.

Solution:

$$\int \Big[C_1 + \tfrac{1}{4}w^2 + 2\int f(w)\,dw\Big]^{-1/2} dw = C_2 \pm \ln x,$$

where C_1 and C_2 are arbitrary constants.

References

Kamke, E., *Differentialgleichungen: Lösungsmethoden und Lösungen, I, Gewöhnliche Differentialgleichungen*, B. G. Teubner, Leipzig, 1977.

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.

