



15. $y''_{xx} = \frac{\psi''_{xx}}{\psi} y + \psi^{-3} f\left(\frac{y}{\psi}\right), \quad \psi = \psi(x).$

The transformation $\xi = \int \frac{dx}{\psi^2}, \quad w = \frac{y}{\psi}$ leads to an autonomous equation of the form 3.1: $w''_{\xi\xi} = f(w).$

Solution:

$$\int \left[C_1 + 2 \int f(w) dw \right]^{-1/2} dw = C_2 \pm \int \frac{dx}{\psi^2(x)},$$

where C_1 and C_2 are arbitrary constants.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.