



22. $y''_{xx} = [e^{\alpha x} f(y) + \alpha] y'_x.$

The substitution $w(y) = e^{-\alpha x} y'_x$ leads to a first-order separable equation: $w'_y = f(y).$

Solution:

$$\int \frac{dy}{F(y) + C_1} = C_2 + \frac{1}{\alpha} e^{\alpha x}, \quad F(y) = \int f(y) dy,$$

where C_1 and C_2 are arbitrary constants.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.