



30. $gy''_{xx} + \frac{1}{2}g'_x y'_x = f(y), \quad g = g(x).$

Integrating yields a first-order separable equation: $g(x)(y'_x)^2 = 2 \int f(y) dy + C_1.$

Solution for $g(x) \geq 0$:

$$\int \left[C_1 + 2 \int f(y) dy \right]^{-1/2} dy = C_2 \pm \int \frac{dx}{\sqrt{g(x)}},$$

where C_1 and C_2 are arbitrary constants.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.