



43. $y''_{xx} + \alpha(y'_x)^2 = [e^{\beta x} f(y) + \beta] y'_x.$

Solution:

$$\int \frac{e^{\alpha y} dy}{F(y) + C_1} = C_2 + \frac{1}{\beta} e^{\beta x}, \quad F(y) = \int e^{\alpha y} f(y) dy,$$

where C_1 and C_2 are arbitrary constants.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.