



45. $y''_{xx} + f(y)(y'_x)^2 - \frac{1}{2}y'_x = e^x g(y).$

The substitution $w(y) = e^{-x}(y'_x)^2$ leads to a first-order linear equation: $w'_y + 2f(y)w = 2g(y).$

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.