8. $(ax^2+bx+c)^4 y'''' = ky.$

The transformation

$$\xi = \int \frac{dx}{ax^2+bx+c}, \qquad w = \frac{y}{(ax^2+bx+c)^{3/2}}$$

leads to a constant coefficient linear equation:

$$w''''_{\xi\xi\xi\xi} - \tfrac{5}{2} D w''_{\xi\xi} + \left(\tfrac{9}{16} D^2 - k\right) w = 0,$$

where $D = b^2 - 4ac$.

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.

