15. \( a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y = 0, \quad a_n \neq 0. \)

**Constant coefficient linear homogeneous differential equation.** The general solution of this equation is determined by the roots of the characteristic equation:

\[
P(\lambda) = 0, \quad \text{where} \quad P(\lambda) = a_n \lambda^n + a_{n-1} \lambda^{n-1} + \cdots + a_1 \lambda + a_0.
\]

The following cases are possible:

1. All roots \( \lambda_1, \lambda_2, \ldots, \lambda_n \) of the characteristic equation are real and distinct. Then the general solution of the homogeneous linear differential equation (1) has the form:

\[
y = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x} + \cdots + C_n e^{\lambda_n x}.
\]

2. There are \( m \) equal real roots \( \lambda_1 = \lambda_2 = \cdots = \lambda_m \) \((m \leq n)\), and the other roots are real and distinct. In this case, the general solution is given by:

\[
y = e^{\lambda_1 x}(C_1 + C_2 x + \cdots + C_m x^{m-1}) + C_{m+1} e^{\lambda_{m+1} x} + C_{m+2} e^{\lambda_{m+2} x} + \cdots + C_n e^{\lambda_n x}.
\]

3. There are \( m \) equal complex conjugate roots \( \lambda = \alpha \pm i \beta \) \((2m \leq n)\), and the other roots are real and distinct. In this case, the general solution is:

\[
y = e^{\alpha x}(C_1 + C_2 x + \cdots + C_m x^{m-1}) + e^{i\beta x}(B_1 x + B_2 x^2 + \cdots + B_m x^{m-1}) + C_{m+1} e^{\lambda_{m+1} x} + C_{m+2} e^{\lambda_{m+2} x} + \cdots + C_n e^{\lambda_n x},
\]

where \( A_1, \ldots, A_m, B_1, \ldots, B_m, C_{m+1}, \ldots, C_n \) are arbitrary constants.

4. In the general case, where there are \( r \) different roots \( \lambda_1, \lambda_2, \ldots, \lambda_r \) of multiplicities \( m_1, m_2, \ldots, m_r \), respectively, the right-hand side of the characteristic equation can be represented as the product

\[
P(\lambda) = (\lambda - \lambda_1)^{m_1} (\lambda - \lambda_2)^{m_2} \cdots (\lambda - \lambda_r)^{m_r},
\]

where \( m_1 + m_2 + \cdots + m_r = n \). The general solution of the original equation is given by the formula:

\[
y = \sum_{k=1}^{r} e^{\alpha k x}(C_{k,0} + C_{k,1} x + \cdots + C_{k,m_k-1} x^{m_k-1}),
\]

where \( C_{k,l} \) are arbitrary constants.

If the characteristic equation has complex conjugate roots, then in the above solution, one should extract the real part on the basis of the relation \( e^{\alpha x} \cos \beta x = e^{\alpha x} (\cos \beta x + \sin \beta x) \).