16. $a_n x^n y_x^{(n)} + a_{n-1} x^{n-1} y_x^{(n-1)} + \cdots + a_1 x y'_x + a_0 y = 0.$

Euler equation.

1°. If all roots λ_k $(k = 1, 2, \ldots, n)$ of the algebraic equation

$$\sum_{\nu=1}^{n} a_\nu \lambda(\lambda-1)\ldots(\lambda-\nu+1) = -a_0$$

are different, the general solution of the Euler equation is given by:

$$y = C_1|x|^{\lambda_1} + C_2|x|^{\lambda_2} + \cdots + C_n|x|^{\lambda_n}.$$

2°. In the general case, the substitution $t = \ln|x|$ leads to a constant coefficient linear equation of the form 4.15 :

$$\sum_{\nu=1}^{n} a_\nu D(D-1)\ldots(D-\nu+1)y = -a_0 y, \qquad \text{where} \quad D = \frac{d}{dx}.$$

References

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