



6. $y''''_{xxxx} = f(y)$.

Autonomous equation. By integrating, we obtain

$$2y'_x y''''_{xxx} - (y''_{xx})^2 = 2 \int f(y) dy + 2C.$$

The substitution $w(y) = |y'_x|^{3/2}$ leads to a second-order equation:

$$w''_{yy} = \frac{3}{2} \left[\int f(y) dy + C \right] w^{-5/3}.$$

Reference

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.