



### 3. Nonlinear Systems of Two Ordinary Differential Equations

#### 3.1. Systems of First-Order Ordinary Differential Equations

1.  $x'_t = x^n F(x, y), \quad y'_t = g(y)F(x, y).$
2.  $x'_t = e^{\lambda x} F(x, y), \quad y'_t = g(y)F(x, y).$
3.  $x'_t = F(x, y), \quad y'_t = G(x, y).$  *Autonomous system of general form.*
4.  $x'_t = f_1(x)g_1(y)\Phi(x, y, t), \quad y'_t = f_2(x)g_2(y)\Phi(x, y, t).$
5.  $x = tx'_t + F(x'_t, y'_t), \quad y = ty'_t + G(x'_t, y'_t).$  *Clairaut system.*

#### 3.2. Systems of Second-Order Ordinary Differential Equations

6.  $x''_{tt} = xf(ax - by) + g(ax - by), \quad y''_{tt} = yf(ax - by) + h(ax - by).$
7.  $x''_{tt} = xf(y/x), \quad y''_{tt} = yg(y/x).$
8.  $x''_{tt} = kxr^{-3}, \quad y''_{tt} = kyr^{-3},$  **where**  $r = \sqrt{x^2 + y^2}.$   
*Equation of motion of a point mass in gravitational field.*
9.  $x''_{tt} = xf(r), \quad y''_{tt} = yf(r),$  **where**  $r = \sqrt{x^2 + y^2}.$   
*Equation of motion of a point mass in central force field.*
10.  $x''_{tt} = xf(x^2 + y^2, y/x) - yg(y/x), \quad y''_{tt} = yf(x^2 + y^2, y/x) + xg(y/x).$
11.  $x''_{tt} = -f(y)g(v)x'_t, \quad y''_{tt} = -f(y)g(v)y'_t - a,$  **where**  $v = \sqrt{(x'_t)^2 + (y'_t)^2}.$   
*Equation of motion of a projectile.*
12.  $x''_{tt} + a(t)x = x^{-3}f(y/x), \quad y''_{tt} + a(t)y = y^{-3}g(y/x).$   
*Generalized Ermakov (Yermakov) system.*
13.  $x''_{tt} = \frac{1}{x^3}F\left(\frac{x}{\varphi(t)}, \frac{y}{\varphi(t)}\right), \quad y''_{tt} = \frac{1}{y^3}G\left(\frac{x}{\varphi(t)}, \frac{y}{\varphi(t)}\right), \quad \varphi(t) = \sqrt{at^2 + bt + c}.$
14.  $x''_{tt} = f(y'_t/x'_t), \quad y''_{tt} = g(y'_t/x'_t).$
15.  $x''_{tt} = x\Phi(x, y, t, x'_t, y'_t), \quad y''_{tt} = y\Phi(x, y, t, x'_t, y'_t).$
16.  $x''_{tt} + x^{-3}f(y/x) = x\Phi(x, y, t, x'_t, y'_t), \quad y''_{tt} + y^{-3}g(y/x) = y\Phi(x, y, t, x'_t, y'_t).$
17.  $x''_{tt} = F(t, tx'_t - x, ty'_t - y), \quad y''_{tt} = G(t, tx'_t - x, ty'_t - y).$
18.  $x''_{tt} = x'_t\Phi(x, y, t, x'_t, y'_t) + f(y), \quad y''_{tt} = -y'_t\Phi(x, y, t, x'_t, y'_t) + g(x).$

$$19. \quad x''_{tt} = ay'_t \Phi(x, y, t, x'_t, y'_t) + f(x), \quad y''_{tt} = bx'_t \Phi(x, y, t, x'_t, y'_t) + g(y).$$

$$20. \quad x''_{tt} = f(y'_t) \Phi(x, y, t, x'_t, y'_t), \quad y''_{tt} = g(x'_t) \Phi(x, y, t, x'_t, y'_t).$$

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The EqWorld website presents extensive information on solutions to various classes of ordinary differential equations, partial differential equations, integral equations, functional equations, and other mathematical equations.

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<http://eqworld.ipmnet.ru/en/solutions/sysode/sode-toc3.pdf>