2. $x'_t = a_1 x + b_1 y + c_1, \quad y'_t = a_2 x + b_2 y + c_2.$

The general solution of this system is given by the sum of its particular solution and the general solution of the corresponding homogeneous system (see equation 1.1).

1°. Let $a_1 b_2 - a_2 b_1 \neq 0$. A particular solution:

$$x = x_0, \qquad y = y_0,$$

where the constants x_0 and y_0 are determined by solving the linear system of equations

$$a_1 x_0 + b_1 y_0 + c_1 = 0, \qquad a_2 x_0 + b_2 y_0 + c_2 = 0.$$

2°. Let $a_1 b_2 - a_2 b_1 = 0$ and $a_1^2 + b_1^2 > 0$. In this case, the system in question becomes

$$x'_t = ax + by + c_1, \qquad y'_t = k(ax + by) + c_2.$$

2.1. If $\sigma = a + bk \neq 0$, a particular solution of the original system is given by

$$x = b\sigma^{-1}(c_1 k - c_2)t - \sigma^{-2}(ac_1 + bc_2), \qquad y = kx + (c_2 - c_1 k)t.$$

2.2. If $\sigma = a + bk = 0$, a particular solution of the original system is given by

$$x = \tfrac{1}{2} b(c_2 - c_1 k)t^2 + c_1 t, \qquad y = kx + (c_2 - c_1 k)t.$$

Reference

Kamke, E., *Differentialgleichungen: Lösungsmethoden und Lösungen, I, Gewöhnliche Differentialgleichungen*, B. G. Teubner, Leipzig, 1977.

