



Systems of Ordinary Differential Equations > Linear Systems of Two Equations

6. $x'_t = f(t)x + g(t)y, \quad y'_t = a[f(t) + ah(t)]x + a[g(t) - h(t)]y.$

Let us multiply the first equation by $-a$ and add it to the second equation to obtain

$$y'_t - ax'_t = -ah(t)(y - ax).$$

Setting $U = y - ax$ and integrating the equation, one obtains

$$y - ax = C_1 \exp \left[-a \int h(t) dt \right], \quad (*)$$

where C_1 is an arbitrary constant. On solving (*) for y and on substituting the resulting expression into the first equation of the system, one arrives at a linear first-order differential equation for x .

Reference

Kamke, E., *Differentialgleichungen: Lösungsmethoden und Lösungen, I, Gewöhnliche Differentialgleichungen*, B. G. Teubner, Leipzig, 1977.