7. $x'_t = f(t)x + g(t)y, \quad y'_t = h(t)x + p(t)y.$

1°. Let us solve the first equation for y and substitute the resulting expression into the second equation to obtain the second-order linear equation

$$g x''_{tt} - (fg + gp + g'_t)x'_t + (fgp - g^2 h + fg'_t - f'_t g)x = 0. \tag{1}$$

This equation is readily integrable if, for example, the following conditions are satisfied:

1) $fgp - g^2 h + fg'_t - f'_t g = 0;$

2) $fgp - g^2 h + fg'_t - f'_t g = ag, \quad fg + gp + g'_t = bg.$

In the former case, equation (1) has a particular solution $u = C = \text{const}$. In the latter case, equation (1) is linear with constant coefficients.

A large number of other solvable cases for equation (1) can be found in the handbooks by E. Kamke (1977) and A. D. Polyanin & V. F. Zaitsev (2003).

2°. Suppose a particular solution,

$$x = x_0(t), \qquad y = y_0(t).$$

of the original system is known. Then the general solution is expressed as

$$x(t) = C_1 x_0(t) + C_2 x_0(t) \int \frac{g(t)F(t)P(t)}{x_0^2(t)}\,dt,$$

$$y(t) = C_1 y_0(t) + C_2 \left[\frac{F(t)P(t)}{x_0(t)} + y_0(t) \int \frac{g(t)F(t)P(t)}{x_0^2(t)}\,dt\right],$$

where C_1 and C_2 are arbitrary constants, and

$$F(t) = \exp\left[\int f(t)\,dt\right], \qquad P(t) = \exp\left[\int p(t)\,dt\right],$$

⊙ *Reference*: A. D. Polyanin, *EqWorld*, 2004 (Private communication, received 23 April 2004).

