



Systems of Ordinary Differential Equations > Linear Systems of Two Equations

9.  $x''_{tt} = a_1x + b_1y + c_1, \quad y''_{tt} = a_2x + b_2y + c_2.$

The general solution of this system is given by the sum of its particular solution and the general solution of the homogeneous system (see equation 1.6).

1°. Let  $a_1b_2 - a_2b_1 \neq 0$ . A particular solution:

$$x = x_0, \quad y = y_0,$$

where the constants  $x_0$  and  $y_0$  are determined by solving the linear algebraic system

$$a_1x_0 + b_1y_0 + c_1 = 0, \quad a_2x_0 + b_2y_0 + c_2 = 0.$$

2°. Let  $a_1b_2 - a_2b_1 = 0$  and  $a_1^2 + b_1^2 > 0$ . In this case, the system in question becomes

$$x''_{tt} = ax + by + c_1, \quad y''_{tt} = k(ax + by) + c_2.$$

2.1. If  $\sigma = a + bk \neq 0$ , the original system has a particular solution

$$x = \frac{1}{2}b\sigma^{-1}(c_1k - c_2)t^2 - \sigma^{-2}(ac_1 + bc_2), \quad y = kx + \frac{1}{2}(c_2 - c_1k)t^2.$$

2.2. If  $\sigma = a + bk = 0$ , the original system has a particular solution

$$x = \frac{1}{24}b(c_2 - c_1k)t^4 + \frac{1}{2}c_1t^2, \quad y = kx + \frac{1}{2}(c_2 - c_1k)t^2.$$

## Reference

**Kamke, E.**, *Differentialgleichungen: Lösungsmethoden und Lösungen, I, Gewöhnliche Differentialgleichungen*, B. G. Teubner, Leipzig, 1977.