



Systems of Ordinary Differential Equations > Linear Systems of Two Equations

10. $x''_{tt} - ay'_t + bx = 0, \quad y''_{tt} + ax'_t + by = 0.$

This equation is used for describing the horizontal motion of a pendulum taking into account the Earth rotation.

Solution with $a^2 + 4b > 0$:

$$x = C_1 \cos(\alpha t) + C_2 \sin(\alpha t) + C_3 \cos(\beta t) + C_4 \sin(\beta t),$$

$$y = -C_1 \sin(\alpha t) + C_2 \cos(\alpha t) - C_3 \sin(\beta t) + C_4 \cos(\beta t),$$

where C_1, \dots, C_4 , and

$$\alpha = \frac{1}{2}a + \frac{1}{2}\sqrt{a^2 + 4b}, \quad \beta = \frac{1}{2}a - \frac{1}{2}\sqrt{a^2 + 4b}.$$

Reference

Kamke, E., *Differentialgleichungen: Lösungsmethoden und Lösungen, I, Gewöhnliche Differentialgleichungen*, B. G. Teubner, Leipzig, 1977.